Reasoning with Semantic Tableau Binary Trees in Description Logic

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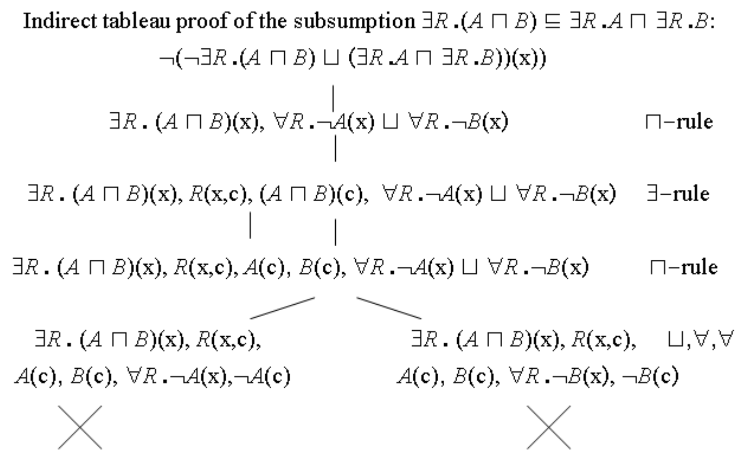
Authors of the Handbook of Description Logic present tableaux algorithms by means of a collection of so-called completion rules intended to generate a completion of ABox with respect to a corresponding TBox of a knowledge base S. An approach we present here is slightly modified. We use a set S of DL formulas of a knowledge base Σ containing DL formulas of ABox and transcriptions of definitions and specifications of TBox into DL formulas. The language used is ALC with role complement. In the cases of direct tableau proofs the tree representation of the tableau proof gives us a possibility of transparent creation of models, especially in the cases of using $\sqcup$ rule (branching of the accompanied tree). An important role play the DL formulas that are logical valid. Those in all interpretations logical valid formulas can become parts of any knowledge base because they don't change its set of models. On the base of the properties of tree representation of indirect tableau proofs of valid formulas we show(see Example, fig.1, 2) a possibility to construct a dual Gentzen-like proof for the sake of direct generation of sentences corresponding to logical consequences of a knowledge base.

In the Example we have constructed a closed tableau tree so the root DL formula is inconsistent. Now let us consider a dual tree: commas between DL formulas represent logical disjunctions of negations of the tableau DL formulas, branching of the tree represent logical conjunctions. Each of the leaf label now contains a complementary pair of DL formulas and represents a logical valid disjunction. It is possible to consider it represents a logical axiom of a Gentzen-like axiomatic system with dual rules to that of the semantic tableau formal system. On the base of closed tableau tree we can create a dual tree representing a Gentzen-like proof. The leaves of the tree are labeled by Gentzen axioms (valid disjunctions of DL formulas. Gentzen-like system rules are dual rules to those of the tableau rules $\sqcap, \sqcup, \forall, \exists$. In the frame of the Gentzen-like system we can proof (see Fig.2) the negated DL formula from Gentzen axioms.

Example (Fig.1):



Gentzen proof (Fig.2):

1. $\neg\exists R.(A \sqcap B)(x), \neg R(x,c), \neg A(c), \neg B(c), \neg\forall R.\neg A(x), A(c)$ **axiom**
2. $\neg\exists R.(A \sqcap B)(x), \neg R(x,c), \neg A(c), \neg B(c), \neg\forall R.\neg B(x), B(c)$ **axiom**
3. $\neg\exists R.(A \sqcap B)(x), \neg R(x,c), \neg(A \sqcap B)(c), \neg(\forall R.\neg A(x) \sqcup \forall R.\neg B)(x)$ **dual $\sqcap$−rule**
4. $\neg\exists R.(A \sqcap B)(x), \neg(\forall R.\neg A(x) \sqcup \forall R.\neg B)(x)$ **dual $\exists$−rule**
5. $(\neg\exists R.(A \sqcap B) \sqcup (\exists R.A \sqcap \exists R.B))(x))$ **dual $\sqcap$−rule, deMorgan**

[1] This work was supported by grant MSM 6198898701 of Czech ministry of education