Inference using Binary, Trivalent and Sigmoid Fuzzy Cognitive Maps

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Abstract

In this paper, we compare the inference capabilities of three different types of Fuzzy Cognitive Maps. A Fuzzy Cognitive Map is a Recurrent Artificial Neural Network that creates models as collections of concepts/neurons and the various causal relations that exist between these concepts/neurons. The three different types of Fuzzy Cognitive Maps that we study are the Binary, the Trivalent and the Sigmoid Fuzzy Cognitive Map, each of them using the corresponding transfer function for their neurons/concepts. Predictions are made by viewing dynamically the consequences of the various imposed scenarios. The prediction making capabilities are examined and presented using a Fuzzy Cognitive Map concerning the Public Health of a city. Conclusions are drawn for the use of the three types of Fuzzy Cognitive Maps for making prediction.

1. Introduction to Fuzzy Cognitive Maps

Fuzzy Cognitive Map (FCM) is a type of Recurrent Artificial Neural Network that has been introduced by Kosko [1], [2] based on Axelord's work on Cognitive Maps [3]. It combines elements of fuzzy logic and artificial neural networks. FCMs create models as collections of concepts and the various causal relations that exist between these concepts. The concepts are represented by neurons and the causal relationships by directed arcs between the neurons. Each arc is accompanied by a weight that defines the type of causal relation between the two concepts/neurons. The sign of the weight determines the positive or negative causal relation between the two concepts/neurons.

Methods for deriving the weights if FCMs can be found in [2,3]. An example of FCM concerning public health is given in Figure 1.

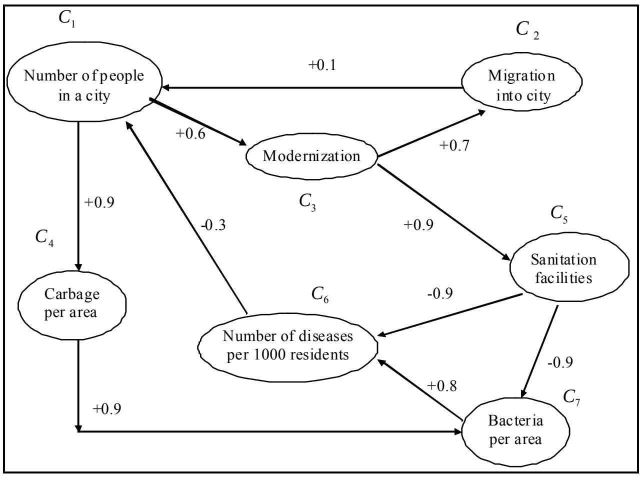

Figure 1: An FCM concerning public health [4]

The structure of an FCM as a Recurrent Artificial Neural Network is that of Figure 2. Each concept/neuron C_i of the FCM graph is accompanied by a number A_i^t that represents the activation level of concept C_i at time step t. If n is the number of FCM's neurons/concepts, then the vector $\mathbf{A^t} = \left[A_1^t, A_2^t, ..., A_n^t\right]$ gives the state of the FCM at time step t.

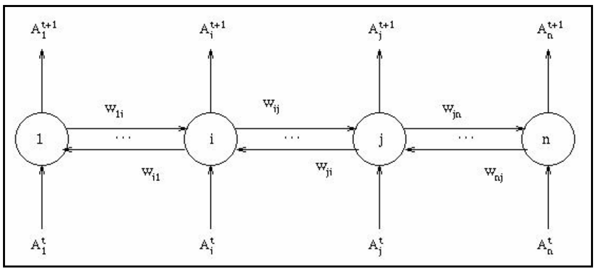

Figure 2: FCM's structure as a Recurrent Artificial Neural Network

The weight matrix W of this Artificial Neural Network is defined by the n×n matrix, where each of its element w_{ij} equals to the weight of the arc that connects concept/neuron C_i with concept/neuron C_j. All elements w_{ii} of the weight matrix W equals to zero, since FCM structure does not allow any direct connection between a concept and itself. All other elements w_{ij} ($i \neq j$) belong to the interval [-1,1]. The FCM is a discrete time system where the activation levels of all the concepts are simultaneously updated, that is the system has synchronous updating [5]. The new state of the system $\mathbf{A^{t+1}} = \left[A_1^{t+1}, A_2^{t+1}, ..., A_n^{t+1}\right]$ is calculated based on the classical operation of the Artificial Neural Networks, that is by evaluating each A_i^{t+1} i=1,...,n, according the following function that McCullock and Pitts [6] have proposed:

$$A_i^{t+1} = f\left(\sum_{j=1}^{n} w_{ji} A_j^t\right) \quad (1)$$

Using weight matrices, we can say that $\mathbf{A^{t+1}} = f\left(\mathbf{A^t W}\right)$. Crucial point to the design of the FCM is the selection of the transfer function *f* of the neurons, which determines the values that activation level A_i^t can take. The transfer functions that are most frequently used are:

1) The sign function [7,8, 9]

$$f_{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases} \quad (2)$$

The use of this function allows the activation level of each neuron to be either 0 or 1, leading to the development of binary FCMs, where each concept is either activated or not activated.

2) The trivalent function [7, 8]

$$f_{tri}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \quad (3)$$

In this case, three state FCMs are created. When the activation level of concept C_i equals 1, it means that this concept increases, when the activation level equals -1, it means that the concept decreases, and when the activation level equals to 0, it means that the concept remain stable.

3) The sigmoid function with saturation levels -1 and 1 [8, 10].

$$f(x) = \tanh(x) \text{ or} \quad (4)$$

$$f(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \quad (5)$$

The activation level can now take any value from the interval [-1,1] and so continuous state FCMs are created.

A binary FCM of n concept has 2^n different states $\left[A_1^t, A_2^t, ..., A_n^t\right]$ and moves at the corners of the $[0,1]^n$ hypercube. A trivalent FCM has 3^n different states and moves at the corners, at the middle of the edges, the center of the sides and the center of the $[-1,1]^n$ hypercube. On the contrary, if the FCM has continuous state values, like the trivalent FCM, then the system has infinite number of different states and moves freely across the whole space of the $[-1,1]^n$ hypercube. In this case, the study of the dynamical behavior of the FCM is the study of the trace of the system to the $[-1,1]^n$ hypercube that the system defines.

2. Dynamical Behavior of FCMs

Conclusions about the model that the FCM represents, are drawn by the study of the dynamical behavior of the FCM system. This resembles the recall phase of the ANN, where the ANN is asked to behave according to the knowledge that has been encoded on the weights of its connections. The inference mechanism of the FCM starts with the initialization of the system [2]. This is done by setting specific values to the activation level of each FCM's concept, based either on the opinions of the experts on the current state of the concepts of the FCM model, or based on a specific scenario of which the consequences we want FCM to predict. After that, the concepts are free to interact. The activation level of each concept influences the other concepts according to the weighted connections that exist between them. This interaction continues until one of the following happens:

1) An equilibrium point is found. In this case, $\exists\, t_0 \in N : A_i^{t+1} = A_i^t, \quad \forall\, t \ge t_0, i = 1,...,n$. The state $\mathbf{A}^{t_0}$ is the final equilibrium point of the system and it is the state to which, the various positive and negative interactions between the neurons/concepts of the FCM, have reached equilibrium.

2) A limit cycle behavior is reached [5, 11]. In this case, $\exists\, t_0, T \in N : A_i^{t+T} = A_i^t, \forall\, t \ge t_0, i = 1,...,n$. The system exhibit a periodic behavior where after a certain number of time steps, equal to the period T of the system, the system reaches the same state.

3) The system exhibit a chaotic behavior [11,12].

The FCM system is deterministic and so, if it reaches a state to which it have been previously, the system will enter a closed orbit which will always repeat. In the case of the binary FCM, since the system has only 2^n different state, it is apparent that after a maximum of 2^n time steps, it will return to a previously visited state. So binary FCM can reach a equilibrium point or a limit cycle behavior (of maximum period 2^n) but never chaotic behavior. For the same reason, a trivalent FCM can never exhibit chaotic behavior but only reach an equilibrium point or a limit cycle behavior with maximum length 3^n. On the contrary, a continuous state FCM can additionally exhibit also chaotic behavior.

The equilibrium point and the limit cycles that the FCM reach correspond to hidden patterns that are encoded in the connections present at the FCM graph [9]. The encoding of these patterns in the FCM structure corresponds to the classical training procedure of the ANNs and is an open research problem.

3. Inference procedure in Binary and Trivalent FCMs

To study the inference procedure of the binary and trivalent FCMs, we will use the FCM of Figure 1. Generally, FCMs reply to "what-if" type of questions. For example, we can assume that there is increase to the city's "Population" (C_1=1), increase to the "Migration" into the city (C_2=1), and increase to its "Modernization" (C_3=1). According to these, the initialization of the system must be done with the vector [1 1 1 0 0 0 0]. Imposing this data to the binary FCM, the dynamical behavior is the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area
0	1	1	1	0
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
Time step t	**Sanitation Facilities**	**Number of diseases per 1000 resident**	**Bacteria per area**	
0	0	0	0	
1	1	0	0	
2	1	0	0	
3	1	0	0	

It can be noticed that the system, after a short period of 3 time steps, it reached an equilibrium point. At that point, there is also increase to the "Garbage per area" and the "Sanitation facilities". An explanation to the above is the following: The increase of the "Population of the city" led to an increase of the "Garbage per area". Moreover, the increase of the "Modernization" led to an increase to the provided "Sanitation Facilities". "Bacteria per area" are not increased, although there is an increase of the "Population of the city", because of the improvement to the "Sanitation Facilities". This also explains why "Number of diseases" is not increased.

It should be stressed that the value 0 at concepts "Bacteria per area" and "Number of diseases" means "no increase". Binary FCM does not allow us to separate the case where a concept is stable, from the case where a concept decreases. A solution comes with use of trivalent FCMs.

Initializing the trivalent FCMs with the same values, the system evolves in the following way:

Time step t	Population of city	Migration into city	Modernization	Garbage per area
0	1	1	1	0
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
Time step t	**Sanitation Facilities**	**Number of diseases per 1000 resident**	**Bacteria per area**	
0	0	0	0	
1	1	0	0	
2	1	-1	0	
3	1	-1	0	
4	1	-1	0	

Once again the system reached quickly an equilibrium point. The final stable state is similar with that of the binary FCM with the difference that concept “Number of diseases” has activation value -1 and so it appears to decrease. So the additional conclusion to those of the binary FCM is that the “Number of diseases” is decreasing while the “Bacteria per area” remain stable.

The increased representing capabilities of the trivalent FCM when compared with the binary FCM, are also illustrated with the following scenario. Let’s assume that due to some external to the FCM systems reasons, there is an increase to the “Sanitation facilities” of the city ($C_6 = 1$). Using the binary FCM, no increase is predicted to any other concept of the FCM, since the system reaches immediately the state A^1=[0 0 0 0 0 0 0], where all concepts are stable and not increasing. On the contrary, the trivalent FCM has the following behavior:

Time step t	Population of city	Migration into city	Modernization	Garbage per area
0	0	0	0	0
1	-1	0	0	0
2	0	0	-1	-1
3	0	-1	0	0
4	-1	0	0	0
5	-1	0	-1	-1
6	-1	-1	-1	-1
7	-1	-1	-1	-1
8	-1	-1	-1	-1
Time step t	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area	
0	0	1	0	
1	0	0	0	
2	0	0	0	
3	-1	0	-1	
4	0	1	1	
5	0	1	0	
6	-1	0	-1	
7	-1	1	0	
8	-1	1	0	

We see that after 8 time steps, the system reached an equilibrium point that is different from that reached by the binary FCM. The trivalent FCM predicts decrease of the “Population of the city”, leading also to a decrease of the “Modernization” of the city, which in turns lead to a decrease of the “Migration into city”. The decrease of the “Modernization” also tends to increase the “Bacteria per area”. But the decrease of the “Population” leads to a decrease of the “Garbage per area” which in turn leads to the increase of the “Bacteria per area”. These two opposite influences neutralize one another and as a result the “Bacteria per area” remain stable. Although this happens, the “Number of diseases” is increasing because of the decrease of the “Sanitation facilities”. All the above can not be predicted by binary FCM since it can not separate the decrease of a concept from the case where the concept remains stable.

Moreover, in trivalent FCMs, we can ask the system to predict the consequences of a decrease of a concept. For example, we can examine the predictions for the case where there is an increase to the “Population of the city” and a decrease to the “Sanitation Facilities”. The evolution of the system is the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area
0	1	0	0	0
1	0	0	1	1
2	-1	1	0	0
3	-1	0	-1	-1
4	1	-1	-1	-1
5	1	-1	1	1
6	-1	1	1	1
7	-1	1	-1	-1
8	1	-1	-1	-1
9	1	-1	1	1
10	-1	1	1	1
11	-1	1	-1	-1
12	1	-1	-1	-1
Time step t	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area	
0	-1	0	0	
1	0	1	1	
2	1	1	1	
3	0	-1	-1	
4	-1	-1	-1	
5	-1	1	0	
6	1	1	1	
7	1	-1	0	
8	-1	-1	-1	
9	-1	1	0	
10	1	1	1	
11	1	-1	0	
12	-1	1	0	

It can be noticed that the state of the trivalent FCM at time step 4, 8 and 12 is the same. The same applies for time steps 5 and 9, time steps 6 and 10 and also time steps 7 and 11. In other words, the system reached a limit cycle behavior with period of 4 time steps which will never stop, unless it is influenced by an external, to the FCM system, factor. The four successive state of the limit cycle are the following:

$\mathbf{A}^1$ =[1 -1-1 -1 -1 -1 -1]
$\mathbf{A}^2$ =[1 -1 1 1 -1 1 0]
$\mathbf{A}^3$ =[-1 1 1 1 1 1 1]
$\mathbf{A}^4$ =[-1 1 -1 -1 1 -1 0]

We can noticed that all concepts in some of the states above increase and in some decrease. Moreover, state $\mathbf{A}^3$ is the opposite of state $\mathbf{A}^1$, and $\mathbf{A}^4$ is the opposite of state $\mathbf{A}^2$. These mean that the city does not reach a stable situation but changes periodically reaching again and again the same or the exact opposite states. It can be concluded that an increase to the "Population of the city" and a decrease to the "Sanitation Facilities" will lead the city to a series of periodic increases and decreases of the FCM's concepts, meaning cycles in the status of the city.

A technique that is frequently used by FCM researchers [7, 9, 13] is that of the restrain of the value of a concept to a certain degree. In this case, we study the consequences of the constant increase or constant decrease of a concept, to all other concepts of the FCM. We can apply this technique in our FCM, assuming that we want to examine what the system predicts for a scenario where the "Sanitation Facilities" of the city constantly decreases and the "Population" of the city initially was increasing. To do that, the concept "Sanitation Facilities" is set to -1 for the whole transition phase of the system towards equilibrium. The transition phase is the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area
0	1	0	0	0
1	0	0	1	1
2	-1	1	0	0
3	-1	0	-1	-1
4	-1	-1	-1	-1
5	-1	-1	-1	-1
Time step t	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area	
0	-1	0	0	
1	-1	1	1	
2	-1	1	1	
3	-1	1	1	
4	-1	1	0	
5	-1	1	0	

As it is shown above, the system reaches an equilibrium point and not a limit cycle. From the state of the system at equilibrium, we can conclude that the constant decrease of the "Sanitation Facilities" of the city leads to a decrease of the "Population", the "Modernization", the "Migration into city" and the "Garbage per area". The decrease of the "Garbage" does not lead to a decrease of the number of "Bacteria" which remains stable because of the influence of the decrease of the "Sanitation Facilities". Only the "Number of diseases per 1000 residents" increases. It should be stressed that this technique resembles the existence of bias to ANN, since having a concept constantly activated to certain level, influences all other concepts that are connected with it, in a similar manner with that of steady bias.

3.1. Studying the Number of different equilibriums that a Binary or Trivalent FCM can reach

Both binary and trivalent FCMs can exhibit a different number of limit cycles or equilibrium points (which also are limit cycles of period 1). To determine all the different limit cycles that such FCMs can reach, we must examine the behavior of the system for all different initial states of the FCM. For the 7-concept FCM of Figure 1, there are 2^7=128 different initial states for the binary FCM and 3^7=2187 for the trivalent FCM. To examine all these case, we developed a computer program written in C programming language, that gives the different limit cycles that the FCM reaches, accompanied with the number of different initial states that reach each of them. For the FCM of Figure 1, the computer program gave the following results:

Behavior	Final State	Length of limit cycle	Number of initial states that reach this final state
A	[0 0 0 0 0 0 0]	1	36
B	[1 1 1 1 1 0 0]	1	6
C	[1 0 0 0 0 0 0] [0 0 1 1 0 0 0] [0 1 0 0 1 0 1]	3	54
D	[0 1 1 1 1 0 1] [1 1 0 0 1 0 0] [1 0 1 1 0 0 0]	3	32

From the above, we see that there are two equilibrium point and two limit cycles of period 3. For the case of the trivalent FCM, the corresponding results are the following:

Behavior	Final State	Length of limit cycle	Number of initial states that reach this final state
A	[0 0 0 0 0 0 0]	1	1
B	[1 -1 -1 -1 -1 1 0]	1	279
C	[1 1 1 1 1 -1 0]	1	279
D	[-1 -1 1 1 -1 -1 -1] [1 1 -1 -1 1 1 1]	2	104
E	[-1 1 -1 -1 1 1 1] [1 -1 1 1 -1 1 0] [1 -1 -1 -1 -1 -1 -1] [-1 -1 -1 -1 1 0]	4	470
F	[1 1 1 1 1 1 1] [1 1 -1-1 1 -1 0] [1 -1 1 1 -1 -1 -1] [-1 1 1 1 1 -1 0]	4	470
G	[-1 1 1 1 1 1 1] [1 -1 1 1 -1 1 0] [1 -1 -1 -1 -1 -1 -1] [-1 1 -1 -1 1 1 -1 0]	4	584

In this case there are three equilibrium points, one limit cycle with period 2 and 3 limit cycles with period 4. We notice that, as we were expecting, trivalent FCM has more complex and wealthy dynamical behavior than that of binary FCM. We should also mention, that the above results are useful, since they shown all the different dynamical behavior that the FCM can exhibit. In this way, we can lead the system to the desired behavior, choosing the correct initial state, leading to a strategic plan for the problem that FCM models. Moreover, we can identify the various limit cycles that the FCM systems can reach and so examine the cycles that for example a city's public health can enter.

4. Inference procedure in Sigmoid FCMs

In the case of the sigmoid FCM, the activation level of the concepts are in the interval [-1,1] and so the initialization of the system can be done with values from the whole interval [-1,1]. Imposing the first scenario that we used for the binary and trivalent FCM, we initialize sigmoid FCM with a "big" increase of city's "Population" (A_1^0=0.8), "medium" increase to "Migration into city" (A_2^0=0.5) and a "small" increase of the "Modernization" (A_3^0=0.3). The system, after a transition phase that is shown in Figure 3, reaches an equilibrium point. The initial and the final state of the system are the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area
0	0.8	0.5	0.3	0
24	0.742	0.888	0.805	0.931
Time step t	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area	
0	0	0	0	
24	0.948	-0.976	-0.037	

We can notice that qualitatively, the conclusion is the same with that of the trivalent FCM, with all concepts to be positively activated, except the "Number of diseases" that is negative and "Bacteria per area" that is close to zero. The advantage of the use of the sigmoid FCM is that now we have an indication of the degree of increase or decrease of the FCM's concepts.

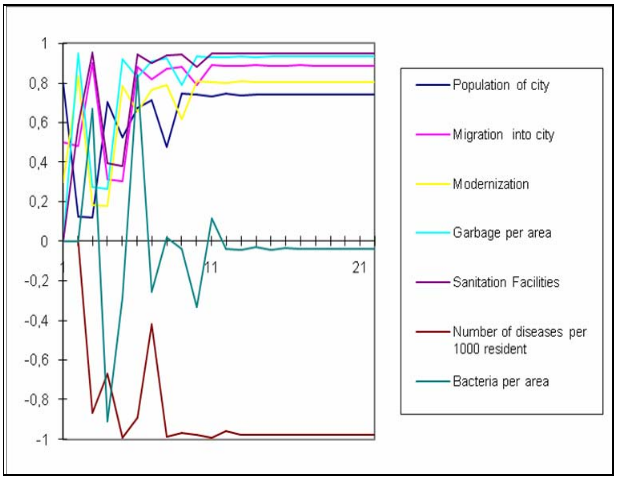

Figure 3: Transition phase of sigmoid FCM having initial state A_1^0=0.8, A_2^0=0.5, A_3^0=0.3.

We can also impose the scenario that we have imposed to trivalent FCM and it reached a limit cycle behavior. In that scenario, we had increase to the "Population of the city" and decrease to the "Sanitation facilities". Since in sigmoid FCMs, also the degree of these increases and decreases can be expressed, we introduce the scenario that initially there is a "big" increase to the "population" (A_1^0 =0.8) and "medium" decrease in the "Sanitation facilities" (A_5^0 =-0.5). The dynamical behavior of the sigmoid FCM is shown in Figure 4, where we can notice that, after 45 time steps, it finally reached an equilibrium point. The initial and final state of the sigmoid FCM are the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area
0	0.8	0	0	0
45	-0.742	-0.888	-0.805	-0.931
Time step t	**Sanitation Facilities**	**Number of diseases per 1000 resident**	**Bacteria per area**	
0	-0.5	0	0	
45	-0.948	0.976	0.037	

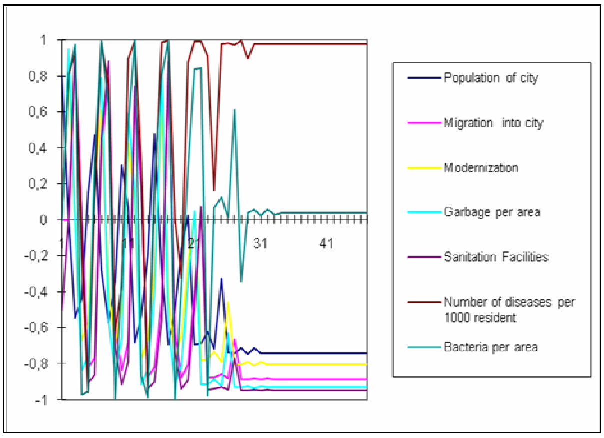

Figure 4: Transition phase of sigmoid FCM having initial state A_1^0=0.8, A_5^0=-0.5.

This dynamical behavior is different from the periodic behavior that the trivalent FCM exhibited. We can also notice that it reaches the exactly opposite equilibrium point from that of Figure 3. This happens because now the weights of the arcs between the neurons of the FCM play a much more important role to that played in binary or trivalent FCMs, since these weights can, in more detail, lead or not to the neutralization of the positive and negative influences a concept/neuron receives from all other neurons.

To illustrate that, we can impose the same scenario of Figure 4, with only one change. The initial decrease of the “Sanitation facilities” from “medium” (A_5^0=-0.5) is changed to “very small” (A_5^0=-0.1). The system exhibits the dynamical behavior of Figure 5.

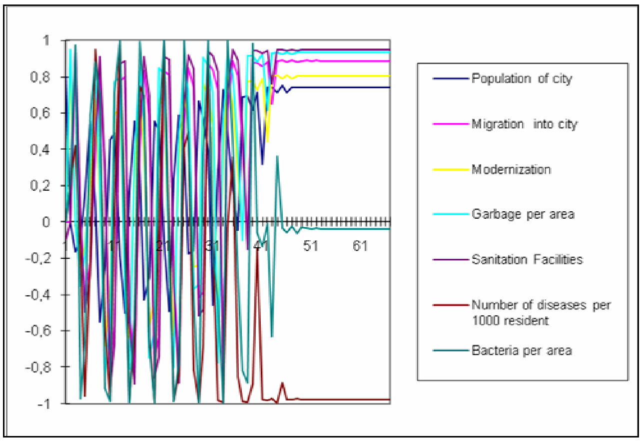

Figure 5: Transition phase of sigmoid FCM having initial state A_1^0=0.8, A_5^0=-0.1.

After a transition phase of 57 time steps, sigmoid FCM reached an equilibrium point. The initial and the final state of the sigmoid FCM are the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area
0	0.8	0	0	0
57	0.742	0.888	0.805	0.931
Time step t	**Sanitation Facilities**	**Number of diseases per 1000 resident**	**Bacteria per area**	
0	-0.1	0	0	
57	0.948	-0.976	-0.037	

It can be noticed that this equilibrium point is the opposite of that reached in the case of Figure 4. This shows that a small change in the initialization of the system can dramatically change its final equilibrium state and also proves that the advanced representing capabilities of sigmoid FCMs leads to better prediction capabilities, with the neuron’s/concept’s activation to be in the whole interval [-1,1].

5. Summary - Conclusions

After a short introduction to Fuzzy Cognitive Maps and their dynamical behavior, the inference procedure for Binary, Trivalent and Sigmoid FCMs is presented using an FCM concerning the Public Health of a city. Various scenarios have been imposed and simulated and conclusions are drawn based on the final state that FCM reached. Comparing the inference capabilities of these three types of FCMs, we draw the following conclusions:

- Binary, Trivalent and Sigmoid FCMs have inference capabilities.

- Binary FCMs can only represent an increase of a concept or represent a stable concept but lack the capability of representing a decrease of a concept.
- Trivalent and Sigmoid FCMs can represent increase or decrease of a concept and also represent a stable concept.
- Binary and Trivalent FCMs can not represent the degree of an increase or a decrease of a concept.
- Sigmoid FCMs, allowing neuron's activation level to be in the whole interval [-1,1], can represent also the degree of an increase or a decrease of a concept.
- In Sigmoid FCMs, small changes to their initial state can lead to a dramatic change to the final state of the FCM.

We can conclude that FCMs can be used as a useful tool for making inference, especially in cases of problems with increased uncertainty and fuzziness.

6. References

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